Language Models Compare Quantities Using Number-specific and Unit-specific Heuristics

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Abstract

Quantities with measurement units, such as *110 cm* and *1.2 m*, require language models (LMs) to combine a numeral with a symbolic unit scale. Here, we study how LMs compare such quantities in controlled settings spanning several unit systems. We find that accuracy degrades near the comparison boundary, where small changes in value determine the correct answer. The resulting errors are systematic: linear surrogate models predict LM preferences from numerical-difference and unit-scale-difference cues, and causal interventions on subspaces aligned with these variables shift model's output. The results suggest that LMs compare quantities through a bag of heuristics over numerals and units, rather than first converting both expressions to an exact shared-scale representation.

mutsumisasaki/LM-Interpret-Units

1 Introduction

Prior work has shown that language models (LMs) encode and compare numerical information in an interpretable manner. Behavioral studies have found magnitude-comparison effects in LM representations (Shah et al., 2023), while mechanistic work has analyzed how models compute greater-than judgments and compare numeric properties (Hanna et al., 2023; El-Shangiti et al., 2025). Other work has studied how numerical values or numeric properties are represented internally (Heinzerling and Inui, 2024; Levy and Geva, 2025). However, quantities in text often appear with measurement units, such as *110 cm*, *1.2 m*, or *3 kg*. Comparing such quantities requires the LM to combine the numeral with the unit scale: *110 cm* has a larger numeral than *1.2 m*, but denotes a smaller length.

Here, we study comparison of quantities with measurement units as a setting where numerical comparison must be performed over heterogeneous expressions. Unlike comparison of bare numbers, each expression contains both a numeral and a symbolic unit, and the larger numeral need not correspond to the larger quantity. Quantity comparison also differs from four arithmetic operations or modular addition, which have been studied in detail (Zhong et al., 2023; Nanda et al., 2023; Stolfo et al., 2023; Ding et al., 2024; Quirke and Barez, 2024; Quirke et al., 2025; Zhang et al., 2024; Zhou et al., 2024). Here, the relevant operation is not a fixed arithmetic function over two numerals: the model must combine each numeral with a symbolic unit and compare the resulting magnitudes on a shared scale. The setting, therefore, gives a controlled test case for extending interpretability work from numerical operations to quantity representations in text.

Our results suggest that LMs may not compare quantities by first converting both expressions into an exact shared-scale representation. Instead, their behavior is better explained as a bag of heuristics over numerical and unit-scale cues. Across LMs and unit systems, accuracy degrades near the comparison boundary. The resulting errors are systematic: linear surrogate models predict LM preferences from numerical-difference and unit-scale-difference features, and causal interventions on subspaces aligned with these variables shift the model's output in the corresponding direction. This parallels recent evidence that LMs solve arithmetic through a bag of heuristics (Nikankin et al., 2025). While the relevant heuristics for the four arithmetic operations include diverse and nontrivial task-solving cues, such as operand ranges, modulo patterns, and result ranges, numerical difference and unit-scale difference emerge as particularly important heuristic cues in our quantity-comparison setting.

By heuristics, we mean comparison cues that are predictive of the model's answer without fully implementing the correct comparison rule. In this setting, such cues include whether the left numeral

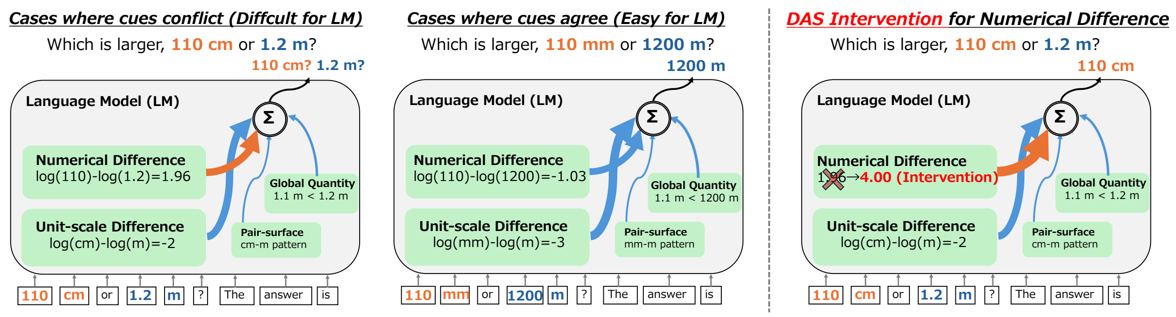

Figure 1: The main illustrations of our work. Language Models (LM) compare quantities with measurement units using heuristic cues mainly on numerical difference and unit-scale difference. The comparison becomes difficult for the LM when these cues vote for opposite sides (left), but is easy when they vote for the same side (center). Causal interventions on representations aligned with these cues shift the LM's output in the corresponding direction (right).

is larger than the right numeral, whether the left unit has a larger scale than the right unit, and thresholded versions of these differences. Aggregating heuristics then means that the model's preference can be predicted from a weighted combination of such cues. This account differs from exact unit conversion: it can explain correct answers when the cues agree, but also predicts systematic errors when numeral and unit-scale cues point in different directions.

We test this account in three steps, summarized in Fig. 1. First, we measure behavioral accuracy across controlled quantity comparisons and show that performance degrades near the comparison boundary. Second, we fit surrogate models over candidate cues and find that numerical-difference and unit-scale-difference features predict LM preferences better than exact shared-scale quantities. Third, we use causal interventions to show that these variables are represented in activation space and can shift the model's output. Together, these results suggest that quantity comparison in LMs is governed by heuristic aggregation over numerals and units, providing a controlled case study of how numerical and symbolic information are combined in language-model representations.

2 Task Formulation and Overall Experimental Setup

To analyze how LMs combine numerical values with symbolic unit information, we study the comparison of quantities with measurement units as a controlled task. This section describes the formulation of the comparison task over quantities with measurement units, as well as the experimental setup shared across this study, including the LMs, prompts, and unit settings.

2.1 Task Formulation

Let D denote a physical dimension, such as length or mass, and let U_D be a chosen set of mutually comparable measurement units for that dimension. A quantity with a measurement unit is specified by a numerical value $r \in [r_{\text{min}}, r_{\text{max}}] \subset \mathbb{R}_{>0}$ and a unit $u \in U_D$, and is written as ru. Given two quantities $q_1 = r_1 u_1$ and $q_2 = r_2 u_2$, where $u_1, u_2 \in U_D$, the LM is asked to decide which is larger:

$$\text{Which is larger, } r_1 u_1 \text{ or } r_2 u_2\text{?} \tag{1}$$

For example, in the comparison "Which is larger, 110 cm or 1.2 m?", we have $D = \text{length}$, $U_D = \{\text{mm}, \text{cm}, \text{m}, \text{km}\}$, $r_1 = 110$, $u_1 = \text{cm}$, $r_2 = 1.2$, and $u_2 = \text{m}$.

For each comparison $q_1 = r_1 u_1$ and $q_2 = r_2 u_2$, the correct answer can be determined by the log-scale difference between the two quantities after accounting for unit scale:

$$\begin{aligned} QM(q_1, q_2) &= \log(r_1) - \log(r_2) \\ &\quad + \log(s_D(u_1)) - \log(s_D(u_2)). \end{aligned} \tag{2}$$

Here, $s_D(u)$ denotes the scale of unit u relative to a base unit for dimension D. We call $QM(q_1, q_2)$ the Quantity Margin. We use this margin throughout the paper to quantify how close a comparison is to the decision boundary and to analyze LM behavior systematically across different numerical values and unit pairs. q_1 is larger when $QM(q_1, q_2) > 0$, and the right quantity q_2 is larger when $QM(q_1, q_2) < 0$.

For the example above, taking meters as the base unit gives $s_D(\text{cm}) = 10^{-2}$ and $s_D(\text{m}) = 1$. Thus, $QM(110\text{cm}, 1.2\text{m}) = \log(110) - \log(1.2) + \log(10^{-2}) - \log(1) \approx -0.04$. The negative margin

Unit setting	Physical dimension D	Unit set U_D
Metric length	length	$\{\mathrm{mm}, \mathrm{cm}, \mathrm{m}, \mathrm{km}\}$
Imperial length	length	$\{\mathrm{inch}, \mathrm{feet}, \mathrm{yard}, \mathrm{mile}\}$
Metric mass	mass	$\{\mathrm{mg}, \mathrm{g}, \mathrm{kg}, \mathrm{t}\}$
Metric-imperial length	length	$\{\mathrm{mm}, \mathrm{cm}, \mathrm{m}, \mathrm{km}, \mathrm{inch}, \mathrm{feet}, \mathrm{yard}, \mathrm{mile}\}$
Metric-imperial mass	mass	$\{\mathrm{g}, \mathrm{kg}, \mathrm{ounce}, \mathrm{pound}\}$

Table 1: Unit settings used in our experiments. Each setting is defined by a physical dimension D and a mutually comparable unit set U_D.

indicates that $1.2\mathrm{m}$ is larger, and its small absolute value indicates that the comparison is close to the decision boundary.

2.2 Overall Experimental Setup

We consider five unit settings, each defined by a physical dimension D and a mutually comparable unit set U_D, as shown in Tbl. 1. For numerical values, we sample r from the range $10^{-3} \leq r < 10^4$ across all unit settings. The first three settings contain units from a single unit system, while the last two combine metric and imperial units, allowing us to test whether the observed behavior is specific to a single unit system or generalizes across both homogeneous and heterogeneous unit comparisons.

We evaluate Qwen3-4B-Base, Qwen3-8B-Base (Yang et al., 2025), and OLMo-3-1025-7B (Olmo et al., 2026). We use multiple prompt templates that ask the LMs to output the larger quantity itself, use greedy decoding, and evaluate predictions by exact match with the correct quantity. Further details on prompt templates, unit scales, and evaluation are provided in § A.

3 Behavioral Observation: LMs Become Less Accurate Near the Boundary

In this section, we examine at the behavioral level under what conditions LMs succeed or fail at comparing quantities with measurement units.

3.1 Experimental Setup

To analyze LMs' behavior systematically, we evaluate their accuracy on the quantity comparison task defined in § 2. We create a controlled dataset by varying both the numerical values and the unit pairs. We group 5,000 comparison examples into 18 bins according to their Quantity Margin, defined in § 2, resulting in 277 or 278 examples per bin. The bins cover negative and positive margins symmetrically, with finer intervals near the decision boundary, e.g., $[-0.2, 0)$ and $[0, 0.2)$. Using the LMs, prompt templates, and unit settings described in § 2, we then evaluate LMs' accuracy within each bin.

3.2 Results

Fig. 2 shows the result for Qwen3-4B-Base across the five unit settings. LM accuracy is strongly organized by the Quantity Margin. Accuracy remains high when the absolute margin is large, but decreases sharply as examples approach the comparison boundary. This indicates that LM comparison difficulty changes systematically with the true quantity difference after accounting for unit scale. In other words, comparisons are easy when the margin is large and become gradually more difficult as the margin approaches zero. The same margin-dependent pattern appears across length and mass comparisons, as well as across homogeneous and heterogeneous unit comparisons. At the same time, heterogeneous metric-imperial comparisons tend to be less accurate than comparisons within a single unit system, with metric length comparisons generally easier than imperial length comparisons. Additional results in § B show that the margin-dependent pattern is robust across LMs, prompt templates, and unit notations, while overall accuracy varies with prompt wording and unit surface form, such as m vs. $meter$.

4 Cue-Combination Hypothesis: Quantity Decisions Depend on Cue Agreement

The behavioral results in Fig. 2 show that comparisons become gradually harder as the Quantity Margin approaches the decision boundary. We interpret this pattern through a cue-combination hypothesis, in which the model combines multiple comparison cues with different weights. Under this view, comparison difficulty depends on how consistently the cues favor the same answer. Far from the boundary, the cues tend to support the same answer and reinforce one another, whereas near the boundary, this support becomes weaker or less consistent.

The cue-combination view raises the question of which cues are available to the model and which

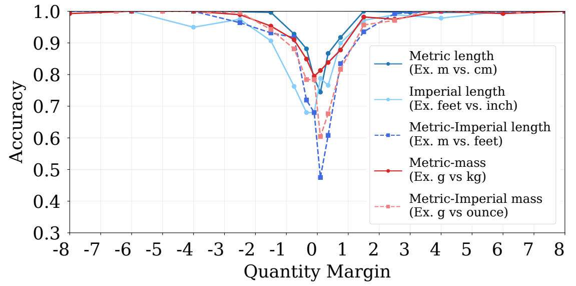

Figure 2: Accuracy of comparisons between quantities with measurement units, grouped by Quantity Margin (Qwen3-4B-Base). Blue and red denote length and mass comparisons, respectively. Dashed curves denote heterogeneous metric-imperial comparisons.

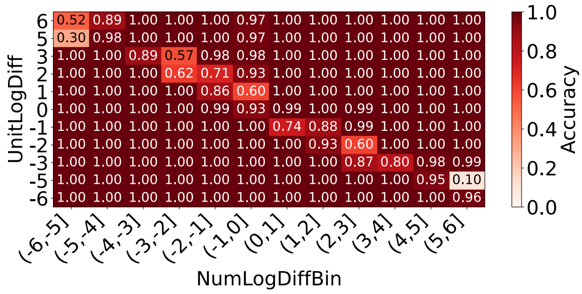

Figure 3: Accuracy of comparisons between quantities with measurement units, grouped by NumLogDiff and UnitLogDiff (Qwen3-4B-Base)

of them actually explain its behavior. We therefore first enumerate candidate cues that could support quantity comparison, and then test their predictive power in the surrogate analysis in § 5.

Several comparison cues are naturally available in this task. The numerical-difference and unit-scale cues are defined as

$$\mathrm{NumLogDiff} = \log r_1 - \log r_2, \tag{3}$$

$$\mathrm{UnitLogDiff} = \log s_D(u_1) - \log s_D(u_2). \tag{4}$$

The magnitude of each quantity on a common scale can also serve as a cue:

$$\mathrm{GlobalLogX} = \log r_1 + \log s_D(u_1), \tag{5}$$

$$\mathrm{GlobalLogY} = \log r_2 + \log s_D(u_2). \tag{6}$$

In addition to these comparison-related quantities, LMs may also rely on more surface-level heuristic cues. For example, digit-based cues can be defined from the appearance of the numerical values, such as

$$\mathrm{firstdigit}(r_1) > n, \tag{7}$$

where n is a threshold. Unit-identity cues may indicate whether a particular unit appears on one side:

$$u_2 \in \{\mathrm{km}, \mathrm{kg}, \mathrm{mile}, \dots\}. \tag{8}$$

Unit-pair cues may depend on familiar ordered pairs of units. For example, in a cm-m pair, the cue may favor the meter expression:

$$(u_1, u_2) = (\mathrm{cm}, \mathrm{m}) \Rightarrow q_2. \tag{9}$$

As a concrete example of cue agreement and conflict, we group examples by NumLogDiff and UnitLogDiff. For metric length comparisons, we bin NumLogDiff over $[-6, 6]$ with width 1, and use the discrete UnitLogDiff values determined by unit pairs, $\{-6, -5, -3, -2, -1, 0, 1, 2, 3, 5, 6\}$. We generate 200 examples for each grid cell and evaluate LM accuracy in each cell. Fig. 3 shows that errors concentrate near the diagonal region where numerical-difference and unit-scale cues cancel each other out, yielding a small Quantity Margin. This pattern illustrates the cue-combination view: comparisons are easy when the relevant cues jointly support the same side, and difficult when they are weak or conflicting. The same qualitative pattern appears across the other unit settings, as shown in § C.

5 Surrogate Analysis: Number and Unit Differences Best Predict LM Behavior

§ 4 introduced a cue-combination hypothesis, in which LM behavior is explained by a weighted combination of heuristic and comparison-related cues. Here, we test this hypothesis by asking which cues best predict the LM's actual behavior.

5.1 Experimental Setup

We use surrogate models to test which candidate quantities and cues explain the LM's comparison behavior. For each comparison example, we instantiate a prompt $\mathrm{Prompt}(q_1, q_2)$, such as "Which is larger, q_1 or q_2? The answer is". We then compare the LM's average log-probability of generating the two candidate answer strings, namely the left quantity a_{q_1} and the right quantity a_{q_2}, as continuations of this prompt:

$$\begin{aligned} m_{\mathrm{LM}}(q_1, q_2) = \bar{\ell}_\theta(a_{q_1} \mid \mathrm{Prompt}(q_1, q_2)) \\ - \bar{\ell}_\theta(a_{q_2} \mid \mathrm{Prompt}(q_1, q_2)). \end{aligned} \tag{10}$$

Here, $\bar{\ell}_\theta(a \mid \mathrm{Prompt})$ denotes the average token log-probability of the full candidate string a when

Surrogate family	Cue variables	Role in analysis
Correct-rule control	Quantity Margin	Baseline for how well the ground-truth comparison rule alone predicts LM behavior.
Global-quantity cues	GlobalLogX, GlobalLogY	Tests whether LM behavior is explained by the two quantities represented on a common scale.
Primitive input components	$\log r_1$, $\log r_2$, $\log s_D(u_1)$, $\log s_D(u_2)$	Tests whether LM behavior is explained by the original numerical and unit components before forming comparison variables.
Number/unit difference cues	NumLogDiff, UnitLogDiff	Tests whether LM behavior is explained by numerical and unit-scale differences.
All heuristic cues	All heuristic cue variables provided in Tbl. 6	Baseline for how well a broad set of heuristic cues can explain LM behavior without directly using the Quantity Margin.
Rule + heuristic cues	Quantity Margin + all heuristic cues	Tests whether heuristic cues add explanatory power beyond the ground-truth comparison rule.

Table 2: Representative surrogate families used in the main analysis. Each surrogate predicts the LM log-probability margin from one family or combination of cue families. The full list of individual features is provided in Tbl. 6, and the full list of surrogate models is provided in Tbl. 7.

appended to the prompt. A positive value indicates that the LM prefers the left quantity, while a negative value indicates that it prefers the right quantity.

Tbl. 2 summarizes the representative surrogate families used in the main analysis. These families are based on a systematic feature design described in § D.1. Starting from the four primitive variables that determine the Quantity Margin, r_1, $s_D(u_1)$, r_2, and $s_D(u_2)$, we construct feature families corresponding to primitive components, number/unit differences, shared-scale global quantities, and the exact comparison rule. We further instantiate these cues as signed, thresholded, and continuous features to capture heuristics at different levels of granularity. The representative families are chosen to compare the main candidate explanations of LM behavior: the ground-truth Quantity Margin, global quantities, primitive numerical and unit components, number/unit differences, and broad heuristic cue sets. The correct-rule control provides a baseline for the true comparison rule, while the heuristic models test whether coarse cues such as signs, thresholds, and identity features explain LM behavior beyond or instead of this rule. Each surrogate predicts the LM log-probability margin from one cue family or a combination of cue families. The full list of individual features is provided in Tbl. 6, and the full list of surrogate models is provided in Tbl. 7.

We evaluate each surrogate model by its held-out coefficient of determination, denoted R^2_{LM}.

However, a high R^2_{LM} can arise simply because a feature set predicts the correct comparison rule. Since the LMs are correct on many examples, features that explain the ground-truth rule may also appear to explain LM behavior. We therefore fit the same surrogate model to the ground-truth Quantity Margin $M(q_1, q_2)$ and report the held-out score as R^2_{Rule}. This score serves as a control for how much each feature set explains the task structure itself.

To quantify how well a feature set explains LM behavior after accounting for this rule-based control, we report a Predictivity score. For the analysis over all examples, we define

$$\text{Predictivity} = R^2_{\text{LM}} \times \frac{R^2_{\text{LM}}}{R^2_{\text{Rule}}}, \qquad (11)$$

which is high when the feature set explains the LM's behavior beyond what is expected from the ground-truth rule alone. We also analyze examples near the decision boundary, where $|QM(q_1, q_2)| \leq 0.2$. As shown in Fig. 2, this is the region where LM accuracy drops most clearly. Under the cue-combination hypothesis, this region is also where different cues are expected to become less consistently aligned. Analyzing these boundary examples therefore lets us test which feature sets explain LM behavior in cue-conflicting cases.

For this boundary-region analysis, R^2_{LM} can be unstable or negative because the examples are intrinsically difficult. We therefore use a difference-based version of the Predictivity score:

$$\text{Predictivity} = R^2_{\text{LM}} - \max(R^2_{\text{Rule}}, 0), \qquad (12)$$

defined only when $R^2_{\text{LM}} > 0$.

Implementation details. We construct a broad surrogate-analysis dataset by sampling examples

Feature set	All examples			$\lvert$Quantity Margin$\rvert \leq 0.2$		
	R^2_{LM}	R^2_{Rule}	Pred↑	R^2_{LM}	R^2_{Rule}	Pred↑
Signed NumLogDiff & UnitLogDiff	0.817	0.823	0.812	0.763	-322.391	0.763
All heuristic features	0.891	0.998	0.796	0.747	-2.753	0.747
Quantity Margin + all heuristics	0.891	1.000	0.794	0.750	1.000	-0.250
NumLogDiff & UnitLogDiff thresholds	0.836	0.997	0.702	0.749	-11.255	0.749
Primitive component thresholds	0.714	0.996	0.512	0.077	-2.559	0.077
NumLogDiff & UnitLogDiff	0.712	1.000	0.507	0.063	1.000	-0.937
GlobalLogX & GlobalLogY	0.692	1.000	0.478	0.035	1.000	-0.965
Quantity Margin only	0.690	1.000	0.476	0.022	1.000	-0.978

Table 3: Representative surrogate results for the Metric length (Ex. m vs. cm) setting in Qwen3-4B-Base. R^2_{LM} measures how well each feature set predicts the LM's log-probability margin, while R^2_{Rule} measures how well it predicts Quantity Margin. Pred denotes Predictivity. For all examples, $\mathrm{Pred} = R^2_{\mathrm{LM}} \times (R^2_{\mathrm{LM}}/R^2_{\mathrm{Rule}})$. For the selected subset (|Quantity Margin| ≤ 0.2), $\mathrm{Pred} = R^2_{\mathrm{LM}} - \max(R^2_{\mathrm{Rule}}, 0)$ when $R^2_{\mathrm{LM}} > 0$, and is left undefined otherwise. Rows are sorted by all-example Pred in descending order. Detailed results for all surrogate models are provided in Tbl. 8.

evenly across quantity-margin bins. We use 20,000 examples for training, 1,000 examples for validation, and 5,000 examples for testing. The LM outputs used as regression targets are obtained with the same LMs, unit settings, prompt templates, and decoding setup as in § 2.2. For each feature set, we fit a ridge regression surrogate model to predict $m_{\mathrm{LM}}(q_1, q_2)$. All features are standardized using the training set statistics. The regularization strength is selected separately for each surrogate model from $\alpha \in \{0.01, 0.1, 1, 10, 100\}$ based on validation R^2, and the selected surrogate model is evaluated on the held-out test set.

5.2 Results

Tbl. 3 shows results for the representative surrogate families summarized in Tbl. 2, using Qwen3-4B-Base on metric length comparisons. Here, signed features use only the direction of a cue, such as whether $\mathrm{NumLogDiff} > 0$ or $\mathrm{UnitLogDiff} > 0$, while threshold features use coarse magnitude information, such as whether a log-scale variable exceeds a threshold n. Other continuous feature sets use the original log-scale values directly.

For the all-example setting, the LM behavior is best explained by heuristic features based on $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$. The highest Predictivity is obtained by signed $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$, and thresholded $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$ also achieve a high score. This indicates that the LM's log-probability margin is especially well captured by coarse cues about which side has the larger numeral and which side has the larger unit scale. By contrast, feature sets based on the Quantity Margin, $\mathrm{GlobalLogX}$ and $\mathrm{GlobalLogY}$, or the primitive input components obtain lower Predictivity, despite their strong ability to explain the ground-truth rule. These results suggest that the LM's comparison behavior is better explained by heuristic cues derived from numerical difference and unit-scale difference.

We next focus on examples near the decision boundary, where $|QM(q_1, q_2)| \leq 0.2$, the region where LM accuracy drops most clearly in Fig. 2. In this region, heuristic features have low R^2_{Rule}, indicating that they do not reliably predict the correct answer when cues conflict. Nevertheless, the LM's behavior is still well predicted by signed and thresholded $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$, while exact-rule and global-quantity feature sets have much lower R^2_{LM}. This suggests that, in the cue-conflicting cases where the LM often fails, it may continue to rely on numerical-difference and unit-scale-difference heuristics.

Overall, the surrogate analysis supports a heuristic account centered on numerical difference and unit-scale difference. Full results across LMs, unit settings, and prompts are provided in § D, where we observe the same results.

6 Causal Analysis: Number and Unit Differences Steer LM Decisions

The surrogate analysis shows that LM behavior is best predicted by heuristic cues based on numerical difference and unit-scale difference. However, surrogate models provide only correlational evidence. They show which cues predict the LM's behavior, but they do not establish whether those cues are internally represented in a way that causally affects the model's output.

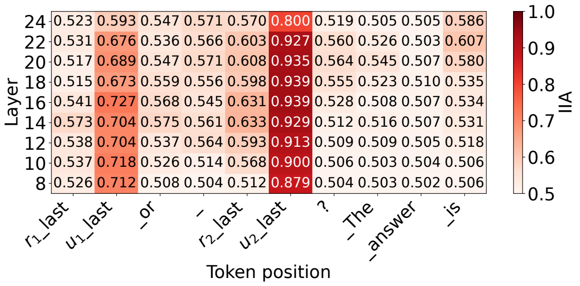

Figure 4: Token-wise DAS intervention accuracy for the NumLogDiff/UnitLogDiff setting in Qwen3-4B-Base on metric length comparisons.

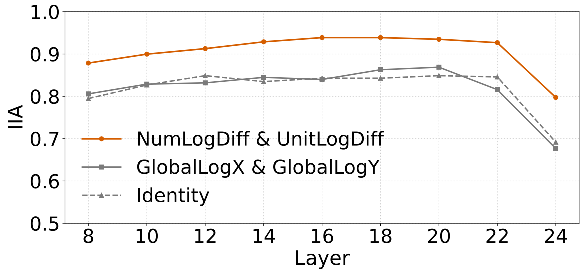

Figure 5: Layer-wise DAS intervention accuracy at the last token of u_2 for Qwen3-4B-Base on Metric length comparisons.

We therefore test whether NumLogDiff and UnitLogDiff are causally active in the LM's internal representations. Using Distributed Alignment Search (DAS) (Geiger et al., 2024), we learn subspaces aligned with these two cues and intervene on them during quantity comparison.

6.1 Experimental Setup

We use Distributed Alignment Search (DAS) to test whether the variables identified by the surrogate analysis are represented in LM activations and causally affect the model's output. DAS learns subspaces of the LM representation aligned with high-level variables, and evaluates whether replacing these subspaces with those from source examples changes the LM's output according to the corresponding high-level counterfactual. We report interchange intervention accuracy (IIA), the fraction of interventions for which the LM's intervened prediction matches the high-level counterfactual label. Full details of the DAS objective and intervention operation are provided in § E.

Our main high-level variables are the numerical difference and unit-scale difference:

$$Z_1 = \mathrm{NumLogDiff}, \quad Z_2 = \mathrm{UnitLogDiff}. \quad (13)$$

For an intervention example $e = (b, s_1, s_2)$, we jointly intervene on both variables by taking NumLogDiff from s_1 and UnitLogDiff from s_2. The high-level counterfactual label is computed as

$$y_{\mathrm{cf}}^{\mathrm{int}}(e) = \mathbb{1}[\mathrm{NumLogDiff}(s_1) + \mathrm{UnitLogDiff}(s_2) > 0]. \quad (14)$$

We compare this setting against two alternative high-level variable sets. The first is a shared-scale global-quantity baseline, which tests whether LM activations are better aligned with the two quantities represented on a common scale:

$$Z_1 = \mathrm{GlobalLogX}, \qquad Z_2 = \mathrm{GlobalLogY}. \quad (15)$$

The second is an identity baseline with four variables corresponding to the primitive log-scale numerical and unit-scale components, which tests whether the intervention effect can be explained by the original numerical and unit-scale components rather than by difference variables:

$$Z_1 = \log r_1, \quad Z_2 = \log s_D(u_1), \quad Z_3 = \log r_2, \quad Z_4 = \log s_D(u_2). \quad (16)$$

For all DAS settings, we fix the total dimensionality of the non-residual intervention subspaces to 1024, allocated evenly across variables. Thus, the two-variable settings use 512 dimensions per variable, while the four-variable identity baseline uses 256 dimensions per variable. As a robustness check, we also vary the total intervention dimensionality to 256, 512, and 2048. We observe the same qualitative pattern, with details provided in § E. Each intervention example consists of a base input and one source input for each high-level variable. We use 20K intervention examples for training and 1K held-out examples for evaluation. We balance output-changing and output-preserving interventions in equal proportions, so the chance-level IIA is 0.5. For all other experimental details, we follow the LMs, prompt templates, and unit settings described in § 2.2.

6.2 Results

Fig. 4 shows the DAS intervention accuracy across layers and token positions for the NumLogDiff/UnitLogDiff setting. The strongest effects concentrate around the last token of u_2. This suggests that information relevant to the comparison is integrated near the end of the second quantity.

We therefore focus on the last token of u_2 for the layerwise analysis below.

Fig. 5 reports layerwise DAS results at the last token of u_2 for Qwen3-4B-Base on metric length comparisons. The $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting achieves the highest IIA across all reported layers, with scores well above the chance level of 0.5 and exceeding 0.9 across many middle layers. This indicates that intervening on the learned subspaces for numerical difference and unit-scale difference can causally steer the LM's comparison output in the direction predicted by the high-level counterfactual.

The two baselines also achieve above-chance IIA, suggesting that global-quantity information and primitive input components are represented to some extent. However, both $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$ and the identity baseline remain consistently below the $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting. This gap suggests that the LM's comparison behavior is more directly controlled by numerical-difference and unit-scale-difference variables than by shared-scale global quantities or primitive input components.

Thus, the numerical difference and unit-scale difference identified in the surrogate analysis are not merely correlated with LM behavior; they are represented in a way that can causally affect the LM's decisions. We observe the same qualitative pattern across other LMs, unit settings, and prompt templates. Full results are provided in § E.

7 Related Work

Prior work (Park et al., 2022; Spokoyny et al., 2022; Göpfert et al., 2022; Xu et al., 2024; Huang et al., 2024; Bui et al., 2025) has examined empirical behavior of LMs on measurement-unit tasks, including basic measurement understanding (Park et al., 2022; Spokoyny et al., 2022), measurement extraction from text (Göpfert et al., 2022), unit-aware chain-of-thought reasoning (Xu et al., 2024), dimension-aware quantitative reasoning (Huang et al., 2024), and generalization across measurement systems (Bui et al., 2025). Building on these behavioral and task-oriented findings, we provide, to our knowledge, the first internal-mechanism analysis of comparing quantities with measurement units.

Prior work has shown that numerical information can be read out from the hidden states of LMs and that this information sometimes causally affects their behavior. Numerical values and attributes have been analyzed through linear directions (Zhu et al., 2025), low-dimensional subspaces (Heinzerling and Inui, 2024; El-Shangiti et al., 2025; AlQuabeh et al., 2026), digit-wise structure (Levy and Geva, 2025), and spiral-like representations (Kantamneni and Tegmark, 2025). For numerical comparison, behavioral work has found magnitude-comparison effects in LMs (Shah et al., 2023), and mechanistic studies have analyzed how models compute greater-than judgments (Hanna et al., 2023). More closely related, Yuchi et al. (2026) show that hidden states encode both the magnitudes of numbers written in different numerical notations, such as standard and exponential notation 5.7×10^2, and which number is larger. While these works focus on numerical values alone, our setting requires LMs to integrate numerals with symbolic unit scales, which introduces an additional compositional step. In addition, we not only find where the answer is encoded, but also how the numerals and units jointly contribute to the LM's behavior.

Regarding how LMs make judgments through heuristic aggregation, Nikankin et al. (2025) argue that, for the four arithmetic operations, LMs solve arithmetic problems by a diverse collection of nontrivial input- and result-pattern heuristics rather than exact algorithms. In contrast, our results suggest that quantity comparison is governed more centrally by the numerical difference and the unit-scale difference.

8 Conclusion

We studied how language models compare quantities with measurement units, such as *110 cm* and *1.2 m*. Across controlled unit settings, LM accuracy decreases near the comparison boundary. Through surrogate analysis, we found that LM preferences are better explained by numerical-difference and unit-scale-difference cues than by exact shared-scale quantities. Through DAS interventions, we further showed that numerical difference and unit-scale difference are represented in activation space and can causally steer model outputs. Overall, our results suggest that LMs compare quantities not by first performing exact unit conversion, but by aggregating heuristics over numerals and unit scales. This provides a controlled case study of how numerical information and symbolic measurement units are combined in language-model representations.

Limitations

While our work shows that language models compare quantities with measurement units using heuristics based on numerical difference and unit-scale difference, it has several limitations.

First, our study focuses on controlled, single-step comparisons between two quantities with measurement units. This design allows us to analyze LM behavior in a controlled setting, but it does not cover more realistic multi-step quantitative reasoning, where unit conversion may be combined with retrieval, arithmetic, or reasoning over longer textual contexts. Extending the analysis to such settings is an important direction for future work.

Second, our surrogate analysis uses linear models to approximate LM behavior from candidate cue features. This choice makes the analysis interpretable and allows us to compare feature families directly, but it may miss nonlinear interactions among cues or more complex heuristic strategies. Similarly, our DAS analysis tests whether selected variables are represented in linear subspaces, rather than fully identifying the circuit components that implement these computations.

Third, extending our analysis to reasoning-oriented models or long chain-of-thought outputs is nontrivial. Such models may distribute comparison-relevant information over many generated tokens, making it less clear where and when unit-comparison information is encoded and used. Our current analysis focuses on short answer generation, where the relevant comparison information is localized around the input tokens. Future work could adapt the proposed analyses to reasoning models and multi-token reasoning traces.

Ethical Considerations

All data created and/or used in this work was synthetically generated. All language models used in this study are publicly available. We strictly adhered to the terms and conditions of each model's license. During the development of code and the writing of this paper, we made use of AI assistants, including large language models. All code snippets and textual content generated with the assistance of such tools were carefully reviewed and revised by the authors to ensure scientific integrity, accuracy, and ethical compliance.

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A Overall Experiment Setup

This section provides additional details on the experimental setup shared across our behavioral, surrogate, and causal analyses. We first describe the prompt templates and unit notations used in the behavioral analysis, and then explain the prompt and notation choices used in the surrogate and DAS analyses.

A.1 Prompt Templates

For the behavioral analysis in § 3, we evaluate LMs under a broad set of prompt designs. The prompts vary along two axes. First, we vary the comparison direction, asking either for the larger quantity or the smaller quantity. Second, we vary the position of the two quantities relative to the comparison phrase. In the postposed design, the two quantities appear after the comparison phrase, as in "Which is larger, q_1 or q_2?". In the proposed design, the two quantities appear before the comparison phrase, as in "Between q_1 or q_2, which is larger?". For mass comparisons, we use "heavier" and "lighter" instead of "larger" and "smaller". The full set of prompt templates is shown in Tbl. 4.

We also vary the surface form of unit expressions. For each unit, we test both a short notation, such as "cm", and a long notation, such as "centimeter". The unit notations are listed in Tbl. 5. This allows us to test whether the observed behavioral patterns depend on a particular surface form of measurement units.

A.2 Choice of Prompt for Main Analyses

In the behavioral analysis, we observe some variation across prompt templates and unit notations. In particular, the postposed design generally yields higher accuracy than the preposed design, and the surface form of the unit notation leads to small differences in performance. However, the main qualitative pattern is stable across these variations. Across prompt templates and unit notations, accuracy changes systematically with the Quantity Margin, and performance decreases near the comparison boundary. Based on this observation, the surrogate analysis in § 5 and the DAS analysis in § 6 use the first larger-comparison prompt in Tbl. 4, with the postposed design and short unit notation. For Qwen3-4B-Base, we also confirm that the second larger-comparison prompt shows the same qualitative pattern.

A.3 Generation and Evaluation

All generations are performed with greedy decoding. For the prompt templates in Tbl. 4, the evaluated models consistently output either q_1 or q_2. This property allows us to use a unified evaluation protocol across the three analyses. For the behavioral analysis, we evaluate predictions by exact match against the correct quantity. For the surrogate analysis, we compare the LM's average log-probability of generating the two candidate answer strings, q_1 and q_2, as continuations of the same prompt. For the DAS analysis, the same binary output structure allows us to train interventions using the correct candidate string as the supervision signal.

B Behavioral Observation: LMs Become Less Accurate Near the Boundary

In § 3, we showed that LM accuracy in quantity comparison is strongly organized by the Quantity Margin. Accuracy remains high when the absolute margin is large, but decreases as the comparison approaches the decision boundary. In this section, we examine the robustness of this gradual margin-dependent pattern and the additional differences that arise across models, prompt templates, and unit notations.

B.1 Results Across Models

Fig. 6 shows the results for Qwen3-4B-Base, Qwen3-8B-Base, and OLMo-3-1025-7B. Across all three models, accuracy changes systematically with the Quantity Margin. Comparisons are easy when the absolute margin is large and become gradually more difficult as the margin approaches zero. This indicates that the boundary-related degradation observed in § 3 is not specific to a single model. The figure also shows differences across unit settings. Heterogeneous metric-imperial comparisons tend to be less accurate than comparisons within a single unit system. For length comparisons, metric comparisons are generally easier than imperial comparisons. Overall, the same margin-dependent pattern is preserved across models and unit settings.

B.2 Effects of Prompt Templates

We next examine the effect of prompt design. As described in Tbl. 4, we vary both the comparison direction and the position of the quantities relative to the comparison phrase. For the comparison direction, we ask either for the larger quantity or for the smaller quantity. For the phrase position, we compare a postposed design, where the quantities q_1 and q_2 appear after the comparison phrase, with a preposed design, where the quantities appear before the comparison phrase. For example, the postposed design uses prompts such as "Which is larger, q_1 or q_2?", whereas the preposed design uses prompts such as "Between q_1 or q_2, which is larger?". Fig. 7 shows the results across these prompt templates for Qwen3-4B-Base. The main margin-dependent pattern is stable across prompt designs. For all prompt templates, comparisons become gradually more difficult as the Quantity Margin approaches zero. At the same time, the prompt wording affects the absolute accuracy. Although the trend is not perfectly consistent across all unit settings, the postposed prompts, shown by solid curves, tend to achieve higher accuracy than the preposed prompts, shown by dashed curves. Similarly, prompts asking for the larger quantity tend to perform better than prompts asking for the smaller quantity, although this effect is also not fully consistent across all settings.

B.3 Effects of Unit Notation

We also examine whether the surface form of units affects LM behavior. As described in Tbl. 5, we compare short unit notation, such as *cm*, with long unit notation, such as *centimeter*. This allows us to test whether the observed behavioral patterns depend on the surface form used to express measurement units. Fig. 8 shows the results for Qwen3-4B-Base. The margin-dependent pattern again remains stable across unit notations. For both short and long unit forms, accuracy decreases as the Quantity Margin approaches zero. However, the choice of notation affects the absolute accuracy. Although the trend is not perfectly consistent across all settings, long-unit notation, shown by red curves, tends to achieve higher accuracy than short-unit notation, shown by blue curves. These results suggest that unit surface forms can influence performance, while the overall boundary-related degradation remains robust.

C Cue-Combination Hypothesis: Quantity Decisions Depend on Cue Agreement

In § 4, we introduced the cue-combination hypothesis, where quantity decisions are explained by the agreement and conflict among comparison cues. As a concrete example, we analyzed the interaction between numerical-difference and unit-scale-difference cues, $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$.

In this section, we show that the same cue-agreement pattern appears across unit settings. Fig. 9 shows LM accuracy grouped by $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$ for each unit setting. Across unit settings, accuracy is high in regions where the two cues support the same answer. In contrast, errors concentrate near the diagonal region where the two cues cancel each other out, yielding a small Quantity Margin. This shows that the cue-conflict pattern discussed in § 4 is not specific to metric length comparisons, but is shared across length, mass, and metric-imperial comparisons.

D Surrogate Analysis: Number and Unit Differences Best Predict LM Behavior

In § 5, we showed that LM comparison behavior is well explained by heuristic cues based on numerical difference and unit-scale difference. In this section, we first describe the full design of the surrogate features. We then provide additional surrogate results and show that the main trend is consistent across unit settings, LMs, and prompt templates. In particular, surrogate models based on signed or thresholded $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$ consistently explain LM behavior well.

D.1 Surrogate Feature Design

Tbl. 6 summarizes the full set of heuristic features considered in our surrogate analysis. We design these features to cover semantically distinct ways of decomposing a quantity comparison while avoiding redundant feature families.

The Quantity Margin in Eq. 2 is determined by four primitive variables: the left numeral r_1, the left unit scale $s_D(u_1)$, the right numeral r_2, and the right unit scale $s_D(u_2)$. We therefore begin with features corresponding to these primitive components. In Tbl. 6, these include Left number log, Right number log, Left unit-scale log, and Right unit-scale log.

We then construct additional feature families by combining subsets of these four primitive variables. One possible decomposition combines three variables on one side and one variable on the other. For example, Left value in right unit together with Right number log represents a comparison between the right numeral and the left quantity converted into the right unit scale. Similarly, the right value in the left unit together with the left number log represents the corresponding comparison in the left unit scale.

Another decomposition splits the four primitive variables into two pairs. The split (r_1, r_2) and $(s_D(u_1), s_D(u_2))$ yields the numerical-difference and unit-scale-difference variables, $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$. This feature family tests whether LM behavior is explained by separately comparing numerals and unit scales. The split $(r_1, s_D(u_1))$ and $(r_2, s_D(u_2))$ yield $\mathrm{GlobalLogX}$ and $\mathrm{GlobalLogY}$, corresponding to the two quantities represented on a common scale. Finally, combining all four variables gives the exact Quantity Margin itself, which we include as the Quantity Margin feature in Tbl. 6. This organization gives a systematic set of semantically distinct feature families, ranging from primitive input components to difference variables, shared-scale quantities, and the exact comparison rule.

To capture heuristic cues at multiple levels of granularity, we instantiate these quantities in several forms. Signed features represent the coarsest cues, such as whether a quantity or difference is positive. Threshold features represent coarse magnitude information, such as whether a log-scale variable exceeds a threshold. Continuous features use the original log-scale values directly.

After all, the resulting list of individual features is shown in Tbl. 6. The surrogate models used in the experiments are then constructed from combinations of these feature families. The full list of surrogate feature sets is provided in Tbl. 7. Among these, Tbl. 2 selects the representative surrogate families that are most important for interpreting the results, and these are used in the main paper.

D.2 Additional Surrogate Results

We next examine whether the main surrogate-analysis finding holds beyond the representative metric-length result in Tbl. 3. Using the Predictivity score defined in § 5, Fig. 10 summarizes the representative surrogate results across LMs and unit settings. Fig. 11 shows the corresponding comparison across prompt templates.

Across these settings, the same qualitative trend appears. Surrogate models based on numerical difference and unit-scale difference, especially signed or thresholded versions of $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$, consistently obtain high Predictivity. This indicates that the LM log-probability margin is well captured by coarse cues about which side has the larger numeral and which side has the larger unit scale. By contrast, feature sets based only on the exact Quantity Margin or on shared-scale quantities, $\mathrm{GlobalLogX}$ and $\mathrm{GlobalLogY}$, tend to obtain lower Predictivity. Although Predictivity provides a compact summary, the raw R^2_{LM} and R^2_{Rule} values are useful for interpreting the results more precisely. We therefore provide representative surrogate tables for each unit setting: Tbl. 3 for metric length, Tbl. 9 for imperial length, Tbl. 10 for metric-imperial length, Tbl. 11 for metric mass, and Tbl. 12 for metric-imperial mass. These tables show that signed and thresholded $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$ features explain LM behavior well across unit settings.

We also provide the full detailed surrogate results for the metric-length setting. While Tbl. 3 reports only the representative surrogate families from Tbl. 2, Tbl. 8 reports results for all surrogate models defined in Tbl. 7. To summarize these detailed results across unit settings, Fig. 12 visualizes Predictivity for all surrogate models using Qwen3-4B-Base. The raw metric-length values are shown in Tbl. 8. Overall, these additional results support the same conclusion as the main analysis: LM quantity-comparison behavior is better explained by numerical-difference and unit-scale-difference heuristics than by exact shared-scale quantities alone.

E Causal Analysis: Number and Unit Differences Steer LM Decisions

In § 6, we used Distributed Alignment Search (DAS) (Geiger et al., 2024) to test whether the variables identified by the surrogate analysis are represented in LM activations and causally affect the model's output. The main result is that interventions on subspaces aligned with $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$ steer the LM's comparison decisions more strongly than interventions on the $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$ baseline or the identity baseline. This suggests that numerical difference and unit-scale difference are not merely correlated with LM behavior, but are represented in a way that can causally affect the model's decisions. In this section, we provide the formal details of DAS and report additional robustness results across models, unit settings, prompt templates, and intervention dimensionalities.

E.1 Distributed Alignment Search

We use Distributed Alignment Search (DAS) to test whether the decomposed cues identified by the surrogate analysis are represented in the LMs and causally affect their output. DAS learns an alignment between high-level causal variables and linear subspaces of a LM's representation by optimizing an interchange intervention objective.

Let $\mathbf{h}(b) \in \mathbb{R}^d$ be the hidden representation of a base input b, and let $\mathbf{h}(s_j) \in \mathbb{R}^d$ be the hidden representation of a source input s_j. For a set of high-level variables $Z_1, \ldots, Z_k$, DAS learns an orthogonal rotation matrix $\mathbf{R} \in \mathbb{R}^{d \times d}$ that decomposes the rotated representation space into orthogonal subspaces:

$$\mathcal{Y} = \mathcal{Y}_0 \oplus \mathcal{Y}_1 \oplus \cdots \oplus \mathcal{Y}_k. \tag{17}$$

Here, $\mathcal{Y}_j$ is aligned with the high-level variable Z_j, and $\mathcal{Y}_0$ is the residual subspace.

A distributed interchange intervention replaces the subspace corresponding to each Z_j in the base representation with the corresponding subspace from the source representation:

$$\begin{aligned}\mathbf{h}^*(b) = \mathbf{R}^{-1}\Bigg(&\mathrm{Proj}_{\mathcal{Y}_0}(\mathbf{R}\mathbf{h}(b)) \\ &+ \sum_{j=1}^{k} \mathrm{Proj}_{\mathcal{Y}_j}(\mathbf{R}\mathbf{h}(s_j))\Bigg).\end{aligned} \tag{18}$$

The intervened representation $\mathbf{h}^*(b)$ is then passed through the remaining LM's layers to obtain the LM's counterfactual output.

For compactness, let $e = (b, s_1, \ldots, s_k)$ denote an intervention example. We train $\mathbf{R}$ so that the LM's counterfactual output matches the counterfactual label predicted after intervening on the corresponding high-level variables. For each intervention example e, we construct a high-level counterfactual by taking the base input and replacing each intermediate variable Z_j with the value computed from the corresponding source input s_j. Let $y_{\mathrm{cf}}^{\mathrm{int}}(e)$ denote this high-level counterfactual label, and let $\mathbf{p}_\theta^{\mathrm{int}}(e)$ denote the LM's output distribution after applying the distributed interchange intervention in Eq. 18. The DAS objective minimizes the cross-entropy loss:

$$\mathcal{L}_{\mathrm{DAS}} = \mathrm{CE}\left(\mathbf{p}_\theta^{\mathrm{int}}(e), y_{\mathrm{cf}}^{\mathrm{int}}(e)\right). \tag{19}$$

We evaluate the learned alignment using interchange intervention accuracy (IIA), the fraction of intervention examples for which the LM's intervened prediction matches the high-level counterfactual label:

$$\mathrm{IIA} = \frac{1}{|\mathcal{D}_{\mathrm{int}}|} \sum_{e \in \mathcal{D}_{\mathrm{int}}} \mathbb{1}\left[\hat{y}_\theta^{\mathrm{int}}(e) = y_{\mathrm{cf}}^{\mathrm{int}}(e)\right], \tag{20}$$

where $\hat{y}_\theta^{\mathrm{int}}(e)$ is the LM's predicted label after intervention. A high IIA indicates that intervening on the learned subspaces changes the LM's output consistently with interventions on the corresponding high-level variables.

E.2 Robustness Across Models, Unit Settings, and Prompts

We first examine whether the main DAS result is robust across LMs. Fig. 13 shows layer-wise IIA at the last token of u_2 for Qwen3-4B-Base, Qwen3-8B-Base, and OLMo-3-1025-7B on metric length comparisons. Across models, the $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting achieves higher IIA than the $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$ and identity baselines across most layers. The absolute IIA differs across models, but the relative ordering is stable: interventions on numerical-difference and unit-scale-difference subspaces have the strongest effect on the model's counterfactual output.

We next evaluate whether this pattern holds across unit settings. Fig. 14 reports the same layer-wise analysis for Qwen3-4B-Base across all unit settings in Tbl. 1. The $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting remains consistently above the two baselines across metric length, imperial length, metric-imperial length, metric mass, and metric-imperial mass settings. This shows that the causal effect of numerical-difference and unit-scale-difference variables is not specific to one unit system or physical dimension.

Finally, we test whether the result depends on prompt wording. Fig. 15 compares the larger-comparison postposed prompt P0 and the larger-comparison preposed prompt P1. In both prompt templates, the $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting obtains higher IIA than the $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$ and identity baselines across layers.

E.3 Robustness to Intervention Dimensionality

The main DAS experiments fix the total dimensionality of the non-residual intervention subspaces to 1024, allocated evenly across variables. To test whether the result depends on this dimensionality choice, we vary the total intervention dimensionality over 256, 512, 1024, and 2048 in the metric length setting. For each DAS setting, the total dimensionality is evenly split among the intervened high-level variables. Thus, the two-variable settings, $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ and $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$, use half of the total dimensionality for each variable, while the four-variable identity baseline uses one quarter of the total dimensionality for each primitive component. Fig. 16 shows IIA as a function of the total intervention dimensionality. Across all dimensionalities, the $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting consistently achieves higher IIA than the $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$ and identity baselines. Moreover, the relative ordering of the three settings remains stable as the dimensionality changes. This indicates that the main causal result is not an artifact of a particular subspace size: numerical-difference and unit-scale-difference variables more directly steer the LM's comparison output across a range of intervention dimensionalities.

Target	Dimension	Design	Prompt template
Larger	Length	Postposed	Which is larger, q_1 or q_2? The answer is
Larger	Length	Preposed	Between q_1 or q_2, which is larger? The answer is
Larger	Mass	Postposed	Which is heavier, q_1 or q_2? The answer is
Larger	Mass	Preposed	Between q_1 or q_2, which is heavier? The answer is
Smaller	Length	Postposed	Which is smaller, q_1 or q_2? The answer is
Smaller	Length	Preposed	Between q_1 or q_2, which is smaller? The answer is
Smaller	Mass	Postposed	Which is lighter, q_1 or q_2? The answer is
Smaller	Mass	Preposed	Between q_1 or q_2, which is lighter? The answer is

Table 4: Prompt templates used in the behavioral analysis. In the postposed design, the two quantities are placed after the comparison phrase. In the preposed design, they are placed before the comparison phrase.

Unit	Short notation	Long notation
Millimeter	mm	millimeter
Centimeter	cm	centimeter
Meter	m	meter
Kilometer	km	kilometer
Inch	in	inch
Foot	ft	foot
Yard	yd	yard
Mile	mi	mile
Milligram	mg	milligram
Gram	g	gram
Kilogram	kg	kilogram
Tonne	t	tonne
Ounce	oz	ounce
Pound	lb	pound

Table 5: Short and long unit notations used in the experiments.

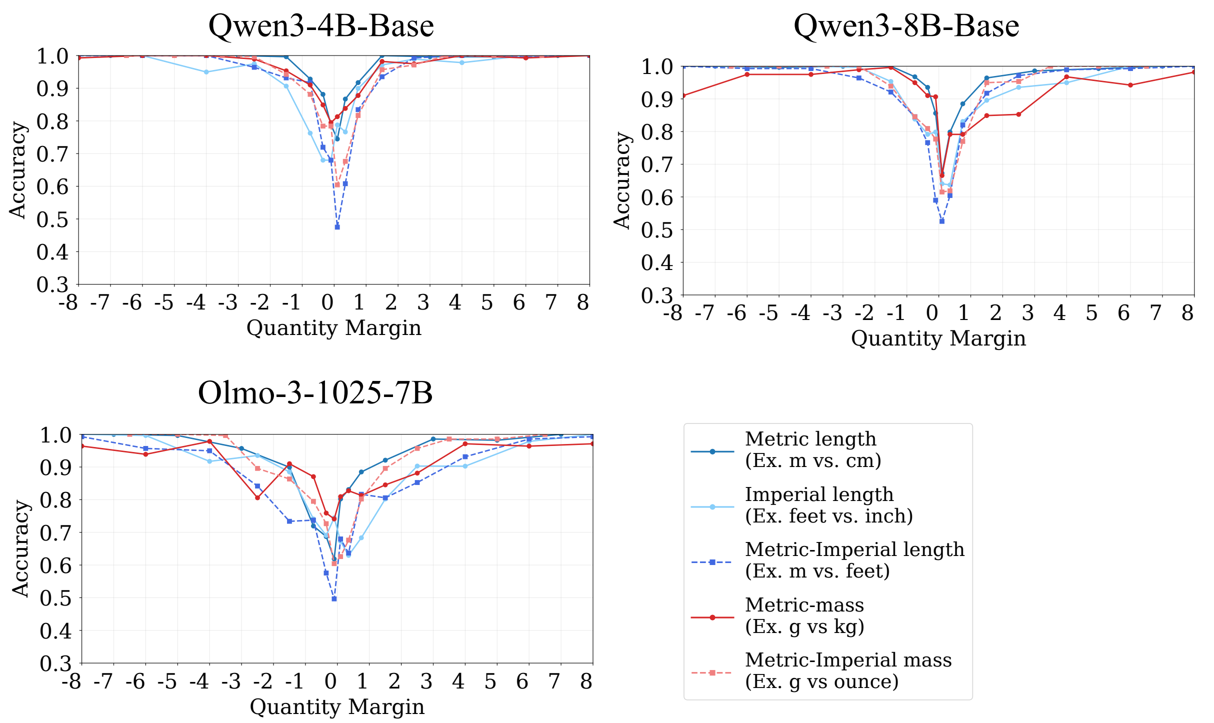

Figure 6: Accuracy of comparisons between quantities with measurement units, grouped by Quantity Margin for Qwen3-4B-Base, Qwen3-8B-Base, and OLMo-3-1025-7B. Blue and red denote length and mass comparisons, respectively. Dashed curves denote heterogeneous Metric-imperial comparisons.

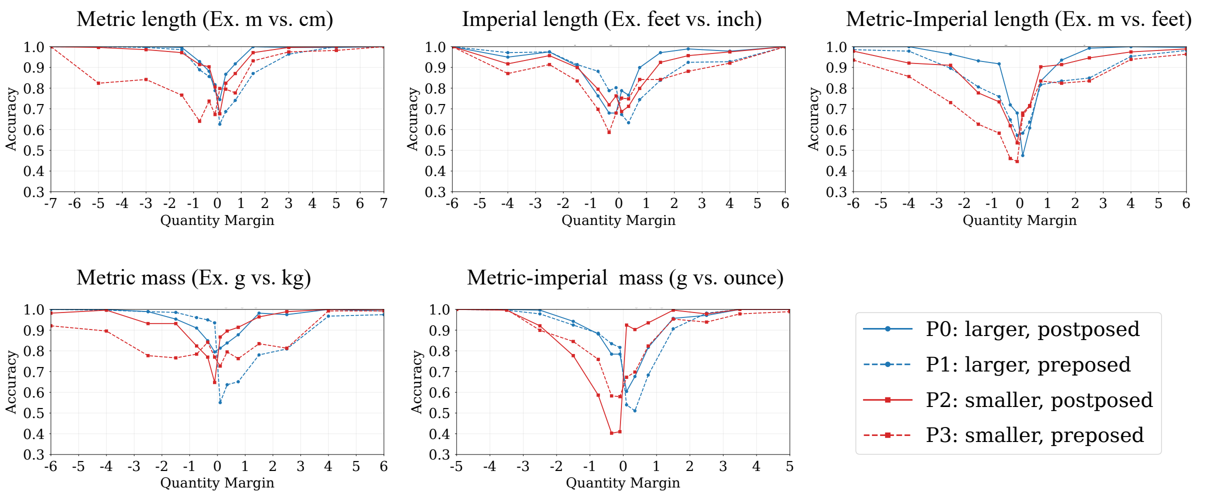

Figure 7: Accuracy of comparisons between quantities with measurement units, grouped by Quantity Margin for Qwen3-4B-Base. Each panel corresponds to a unit setting. Curves compare four prompt designs: larger-postposed, larger-preposed, smaller-postposed, and smaller-preposed.

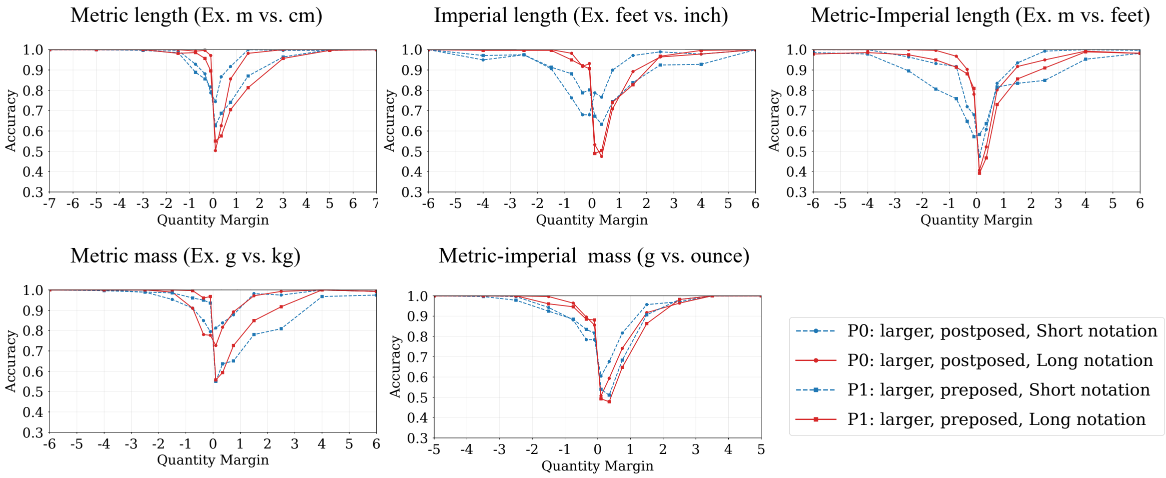

Figure 8: Accuracy of comparisons between quantities with measurement units, grouped by Quantity Margin for Qwen3-4B-Base. Each panel corresponds to a unit setting. Curves compare short-unit notation, such as *cm*, with long-unit notation, such as *centimeter*, across prompt designs.

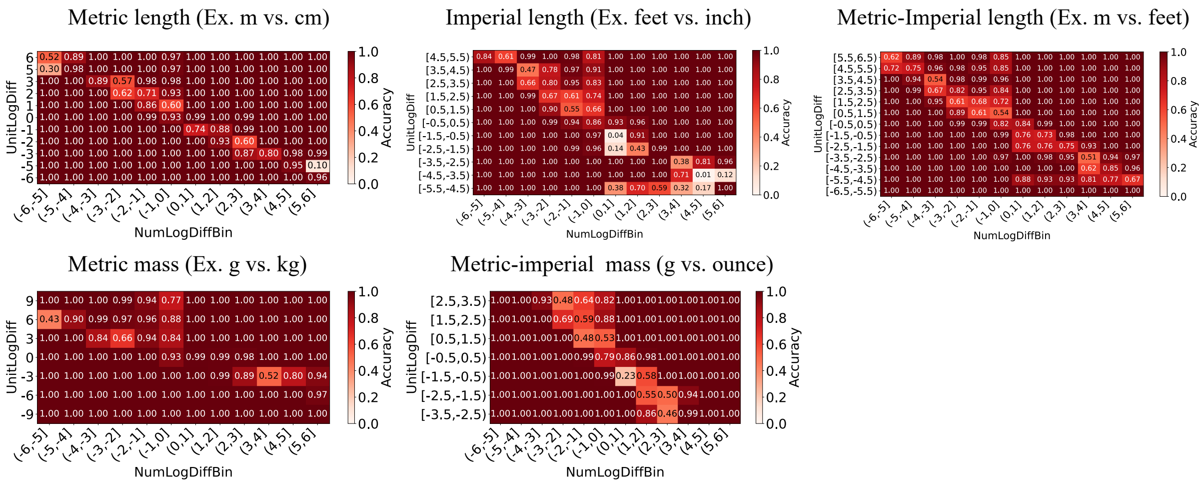

Figure 9: Accuracy of comparisons between quantities with measurement units, grouped by NumLogDiff and UnitLogDiff (Qwen3-4B-Base).

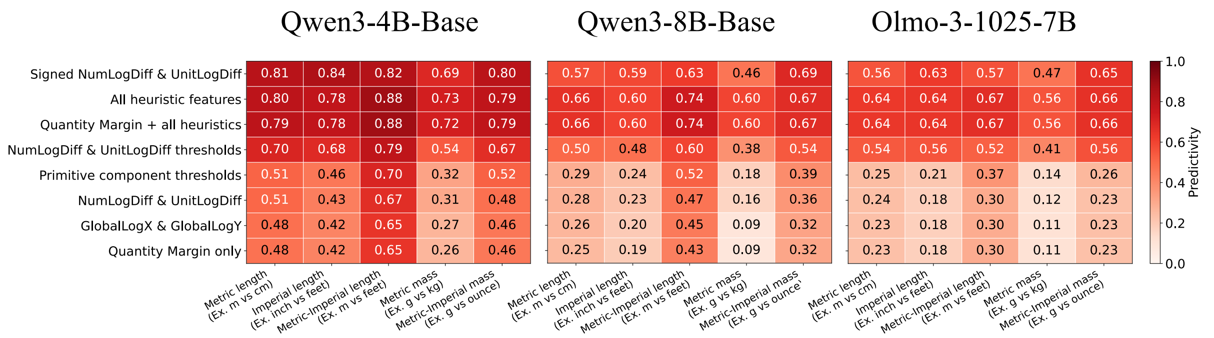

Figure 10: Surrogate Predictivity (Pred) across all unit settings in Tbl. 1 for Qwen3-4B-Base, Qwen3-8B-Base, and OLMo-3-1025-7B. Rows correspond to representative surrogate feature sets, and columns correspond to unit settings. $\text{Pred} = R^2_{\text{LM}} \times (R^2_{\text{LM}}/R^2_{\text{Rule}})$, where R^2_{LM} measures prediction of the LM log-probability margin and R^2_{Rule} measures prediction of Quantity Margin. Higher Pred means that the feature set better predicts LM behavior and is not simply fitting the ground-truth comparison rule.

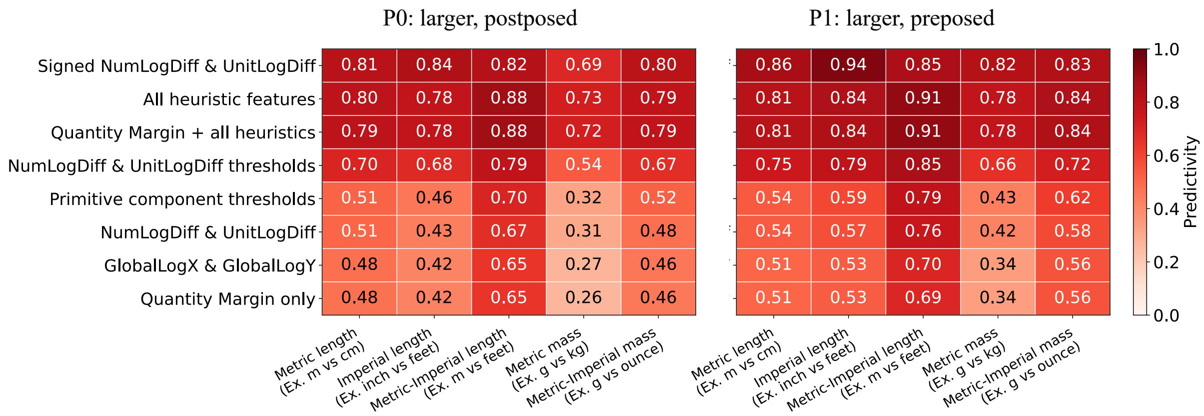

Figure 11: Surrogate Predictivity (Pred) across all unit settings in Tbl. 1 for Qwen3-4B-Base under two prompt templates from Tbl. 4. P0 is the larger-comparison postposed prompt, where the quantities appear after the comparison phrase, e.g., "Which is larger, q_1 or q_2?" P1 is the larger-comparison preposed prompt, where the quantities appear before the comparison phrase, e.g., "Between q_1 or q_2, which is larger?" Rows correspond to representative surrogate feature sets, and columns correspond to unit settings. $\mathrm{Pred} = R^2_{\mathrm{LM}} \times (R^2_{\mathrm{LM}}/R^2_{\mathrm{Rule}})$, where R^2_{LM} measures prediction of the LM log-probability margin and R^2_{Rule} measures prediction of Quantity Margin. Higher Pred means that the feature set better predicts LM behavior and is not simply fitting the ground-truth comparison rule.

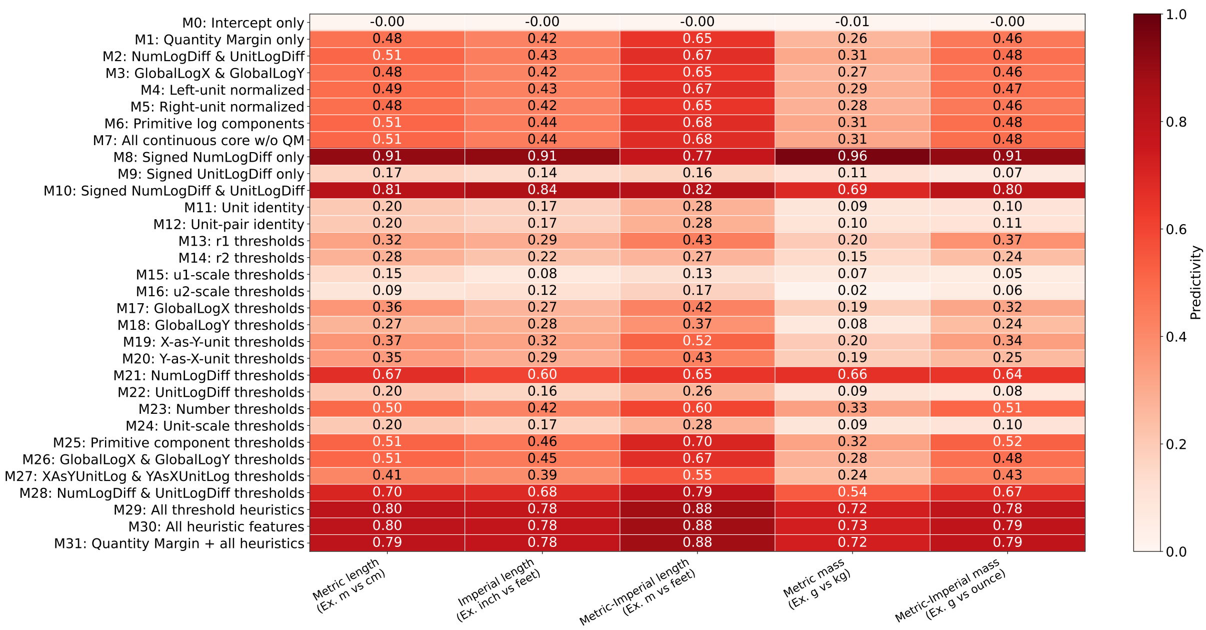

Figure 12: Surrogate Predictivity (Pred) across all unit settings in Tbl. 1 for Qwen3-4B-Base. Rows correspond to all feature sets in Tbl. 7, and columns correspond to unit settings. $\mathrm{Pred} = R^2_{\mathrm{LM}} \times (R^2_{\mathrm{LM}}/R^2_{\mathrm{Rule}})$, where R^2_{LM} measures prediction of the LM log-probability margin and R^2_{Rule} measures prediction of Quantity Margin. Higher Pred means that the feature set better predicts LM behavior and is not simply fitting the ground-truth rule.

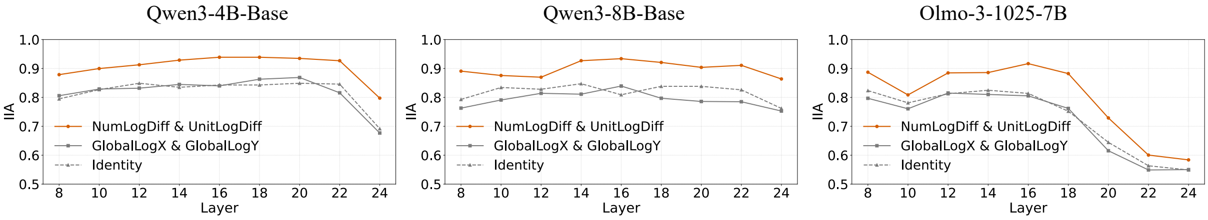

Figure 13: Layer-wise DAS intervention accuracy at the last token of u2 for Qwen3-4B-Base, Qwen3-8B-Base, and Olmo-3-1025-7B on Metric length comparisons.

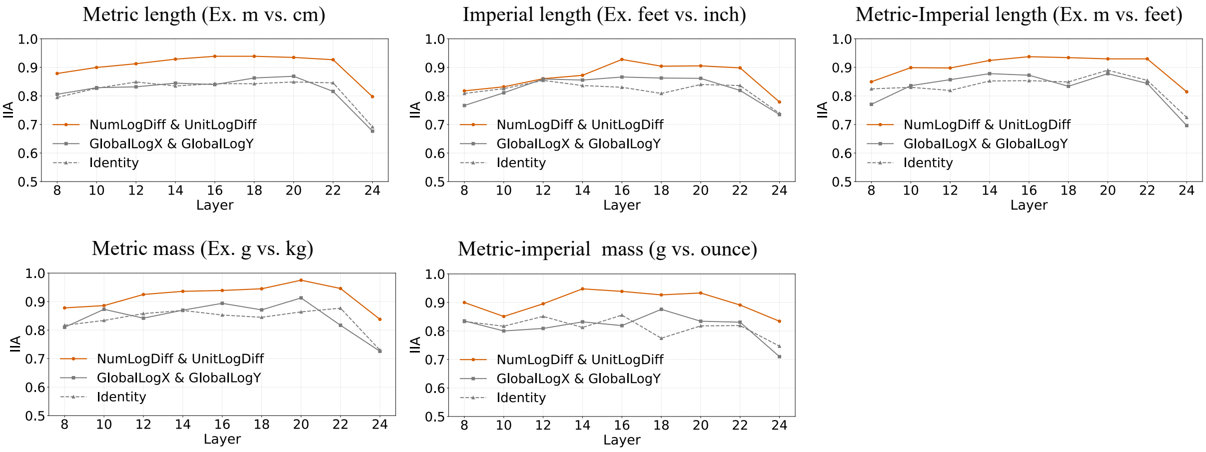

Figure 14: Layer-wise DAS intervention accuracy at the last token of u_2 for Qwen3-4B-Base across all unit settings in Tbl. 1.

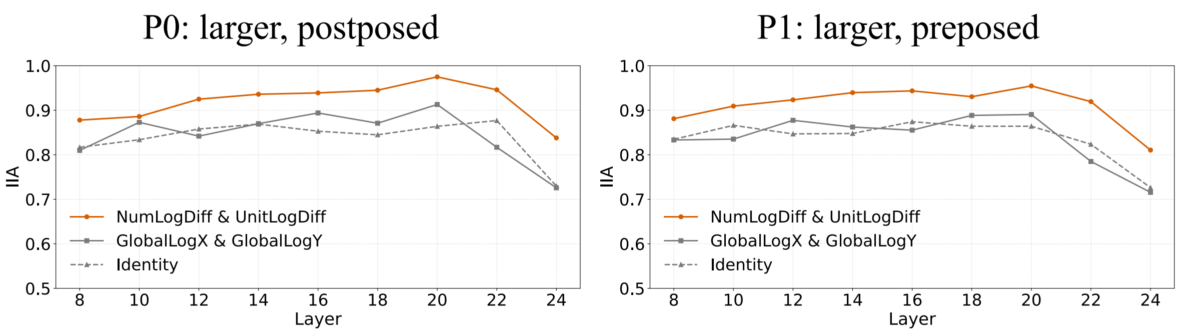

Figure 15: Layer-wise DAS intervention accuracy at the last token of u_2 for Qwen3-4B-Base across Metric-mass settings under two prompt templates. P0 is the larger-comparison postposed prompt, where the quantities appear after the comparison phrase, e.g., "Which is larger, q_1 or q_2?" P1 is the larger-comparison preposed prompt, where the quantities appear before the comparison phrase, e.g., "Between q_1 or q_2, which is larger?"

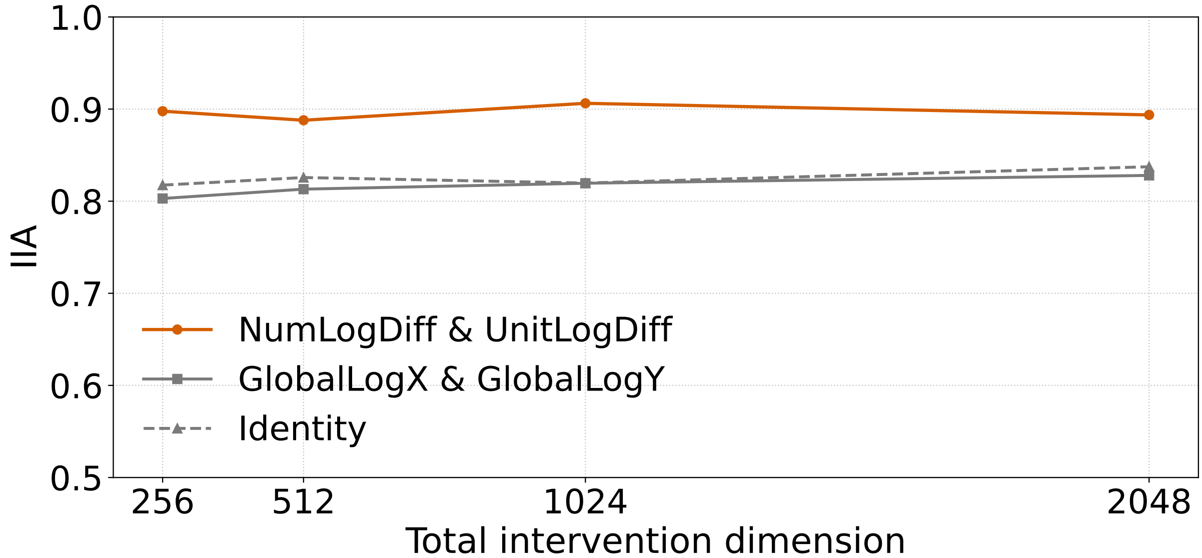

Figure 16: DAS intervention accuracy for Qwen3-4B-Base on the metric length setting as the total dimensionality of the non-residual intervention subspaces is varied. The x-axis shows the total intervention dimensionality, which is evenly allocated across variables within each DAS setting. The y-axis reports interchange intervention accuracy (IIA) at the last token of u_2, averaged over the selected layers from 8 to 24 in steps of 2. Across dimensionalities, the $\mathrm{NumLogDiff}/\mathrm{UnitLogDiff}$ setting consistently achieves higher IIA than the $\mathrm{GlobalLogX}/\mathrm{GlobalLogY}$ and identity baselines.

Feature group	Feature	Definition / Interpretation
Continuous quantity features		
Left number log	F_{1NumLog}	$\log_{10} r_1$.
Right number log	F_{2NumLog}	$\log_{10} r_2$.
Left unit-scale log	F_{1UnitLog}	$\log_{10} s_D(u_1)$.
Right unit-scale log	F_{2UnitLog}	$\log_{10} s_D(u_2)$.
Number-log difference	$F_{\text{NumLogDiff}}$	$\log_{10} r_1 - \log_{10} r_2$.
Unit-log difference	$F_{\text{UnitLogDiff}}$	$\log_{10} s_D(u_1) - \log_{10} s_D(u_2)$.
Left global log	$F_{\text{GlobalLogX}}$	$\log_{10} r_1 + \log_{10} s_D(u_1)$.
Right global log	$F_{\text{GlobalLogY}}$	$\log_{10} r_2 + \log_{10} s_D(u_2)$.
Quantity Margin	$F_{\text{QuantityMargin}}$	$QM(q_1, q_2)$.
Left value in right unit	$F_{\text{XAsYUnitLog}}$	$\log_{10} r_1 + \log_{10} s_D(u_1) - \log_{10} s_D(u_2)$.
Right value in left unit	$F_{\text{YAsXUnitLog}}$	$\log_{10} r_2 + \log_{10} s_D(u_2) - \log_{10} s_D(u_1)$.
Signed comparison heuristics		
Number-log comparison	$H^{\pm}_{\text{numlogdiff}}$	$\text{sign}(\log_{10} r_1 - \log_{10} r_2)$.
Unit-scale comparison	$H^{\pm}_{\text{unitdiff}}$	$\text{sign}(\log_{10} s_D(u_1) - \log_{10} s_D(u_2))$.
Number threshold heuristics		
Left-number threshold	$H_{r_1>10^n}$	$\mathbf{1}[\log_{10} r_1 > n]$, where $n \in \{-3,-2,-1,0,1,2,3\}$.
Right-number threshold	$H_{r_2>10^n}$	$\mathbf{1}[\log_{10} r_2 > n]$, where $n \in \{-3,-2,-1,0,1,2,3\}$.
Unit-scale threshold heuristics		
Left-unit threshold	$H_{s_D(u_1)>10^n}$	$\mathbf{1}[\log_{10} s_D(u_1) > n]$.
Right-unit threshold	$H_{s_D(u_2)>10^n}$	$\mathbf{1}[\log_{10} s_D(u_2) > n]$.
Global quantity threshold heuristics		
Left-global threshold	$H_{\text{GlobalLogX}>n}$	$\mathbf{1}[\log_{10} r_1 + \log_{10} s_D(u_1) > n]$.
Right-global threshold	$H_{\text{GlobalLogY}>n}$	$\mathbf{1}[\log_{10} r_2 + \log_{10} s_D(u_2) > n]$.
Unit-normalized threshold heuristics		
Left as right-unit threshold	$H_{\text{XAsYUnitLog}>n}$	$\mathbf{1}[\log_{10} r_1 + \log_{10} s_D(u_1) - \log_{10} s_D(u_2) > n]$.
Right as left-unit threshold	$H_{\text{YAsXUnitLog}>n}$	$\mathbf{1}[\log_{10} r_2 + \log_{10} s_D(u_2) - \log_{10} s_D(u_1) > n]$.
Difference threshold heuristics		
Number-log-difference threshold	$H_{\text{NumLogDiff}>n}$	$\mathbf{1}[\log_{10} r_1 - \log_{10} r_2 > n]$.
Unit-log-difference threshold	$H_{\text{UnitLogDiff}>n}$	$\mathbf{1}[\log_{10} s_D(u_1) - \log_{10} s_D(u_2) > n]$.
Unit identity heuristics		
Left-unit identity	$H_{u_1=u}$	$\mathbf{1}[u_1 = u]$.
Right-unit identity	$H_{u_2=u}$	$\mathbf{1}[u_2 = u]$.
Unit-pair heuristics		
Unit-pair identity	$H_{(u_1,u_2)=(u,v)}$	$\mathbf{1}[(u_1, u_2) = (u, v)]$.

Table 6: Continuous and heuristic features used in the surrogate models. Here, $q_1 = r_1 u_1$ and $q_2 = r_2 u_2$, and $s_D(u)$ denotes the scale of unit u relative to the base unit for physical dimension D. The continuous features preserve log-scale magnitudes and differences, while the heuristic features encode signed comparisons, threshold indicators, and unit-identity information. $F_{\text{QuantityMargin}}$ corresponds to the correct comparison rule and is treated as a control feature rather than a surface heuristic. Superscript $\pm$ denotes signed comparison heuristics.

Model	Feature set	Included features
M0	Intercept only	Constant baseline with no input feature.
Correct-rule control		
M1	Quantity Margin only	$QM(q_1, q_2)$.
Continuous and algorithmic representations		
M2	NumLogDiff & UnitLogDiff	Continuous difference features: $\mathrm{NumLogDiff}$ and $\mathrm{UnitLogDiff}$.
M3	GlobalLogX & GlobalLogY	Continuous global-magnitude features: $\mathrm{GlobalLogX}$ and $\mathrm{GlobalLogY}$.
M4	Left-unit normalized	Left-unit comparison features: $\log_{10} r_1$ and $\mathrm{YAsXUnitLog}$.
M5	Right-unit normalized	Right-unit comparison features: $\mathrm{XAsYUnitLog}$ and $\log_{10} r_2$.
M6	r_1, r_2, u_1, u_2 logs	Primitive log-space variables: $\log_{10} r_1$, $\log_{10} r_2$, $\log_{10} s_D(u_1)$, and $\log_{10} s_D(u_2)$.
M7	All continuous core w/o Quantity Margin	All continuous variables except direct $QM(q_1, q_2)$.
Simple comparison heuristics		
M8	Signed NumLogDiff only	Signed number-comparison heuristic: $H^{\pm}_{\mathrm{numlogdiff}} = \mathrm{sign}(\mathrm{NumLogDiff})$.
M9	Signed UnitLogDiff only	Signed unit-scale-comparison heuristic: $H^{\pm}_{\mathrm{unitdiff}} = \mathrm{sign}(\mathrm{UnitLogDiff})$.
M10	Signed NumLogDiff & UnitLogDiff	Signed comparison heuristics: $H^{\pm}_{\mathrm{numlogdiff}}$ and $H^{\pm}_{\mathrm{unitdiff}}$.
Identity heuristic families		
M11	Unit identity	One-hot indicators for u_1 and u_2.
M12	Unit-pair identity	One-hot indicators for ordered unit pairs (u_1, u_2).
Single-variable threshold heuristic families		
M13	r_1 thresholds	Threshold indicators over r_1: $H_{r_1>10^n}$.
M14	r_2 thresholds	Threshold indicators over r_2: $H_{r_2>10^n}$.
M15	u_1-scale thresholds	Threshold indicators over the left unit scale: $H_{s_D(u_1)>10^n}$, equivalently $\mathbf{1}[\log_{10} s_D(u_1) > n]$.
M16	u_2-scale thresholds	Threshold indicators over the right unit scale: $H_{s_D(u_2)>10^n}$, equivalently $\mathbf{1}[\log_{10} s_D(u_2) > n]$.
M17	GlobalLogX thresholds	Threshold indicators over GlobalLogX.
M18	GlobalLogY thresholds	Threshold indicators over GlobalLogY.
M19	X-as-Y-unit thresholds	Threshold indicators over $\mathrm{XAsYUnitLog}$.
M20	Y-as-X-unit thresholds	Threshold indicators over $\mathrm{YAsXUnitLog}$.
M21	NumLogDiff thresholds	Threshold indicators over $\mathrm{NumLogDiff}$.
M22	UnitLogDiff thresholds	Threshold indicators over $\mathrm{UnitLogDiff}$.
Grouped threshold heuristic families		
M23	Number thresholds	Both r_1- and r_2-threshold features.
M24	Unit-scale thresholds	Both $s_D(u_1)$- and $s_D(u_2)$-threshold features.
M25	Primitive component thresholds	Threshold features over r_1, r_2, $s_D(u_1)$, and $s_D(u_2)$.
M26	GlobalLogX & GlobalLogY thresholds	Both global-magnitude threshold families.
M27	XAsYUnitLog & YAsXUnitLog thresholds	Both unit-normalized threshold families.
M28	NumLogDiff & UnitLogDiff thresholds	Threshold features over the two difference variables.
M29	All threshold heuristics	All threshold features from M13–M22.
Combined heuristic models		
M30	All heuristic features	Signed comparison heuristics from M10, identity heuristics from M11–M12, and all threshold heuristics from M29.
M31	Quantity Margin + all heuristics	$QM(q_1, q_2)$ plus all heuristic features from M30.

Table 7: Feature sets for the surrogate models. The response variable is the LM's mean log-probability margin between the left and right candidate answers. Here, $q_1 = r_1 u_1$, $q_2 = r_2 u_2$, and $s_D(u)$ denotes the scale of unit u relative to the base unit for physical dimension D.

Model	Feature set	All examples			$\lvert$Quantity Margin$\rvert \leq 0.2$		
		R^2_{LM}	R^2_{Rule}	Pred	R^2_{LM}	R^2_{Rule}	Pred
M8	Signed NumLogDiff only	0.595	0.390	0.908	0.417	-953.511	0.417
M10	Signed NumLogDiff & UnitLogDiff	0.817	0.823	0.812	0.763	-322.391	0.763
M30	All heuristic features	0.891	0.998	0.796	0.747	-2.753	0.747
M29	All threshold heuristics	0.891	0.998	0.796	0.747	-2.700	0.747
M31	Quantity Margin + all heuristics	0.891	1.000	0.794	0.750	1.000	-0.250
M28	NumLogDiff & UnitLogDiff thresholds	0.836	0.997	0.702	0.749	-11.255	0.749
M21	NumLogDiff thresholds	0.643	0.618	0.668	0.345	-753.872	0.345
M25	Primitive component thresholds	0.714	0.996	0.512	0.077	-2.559	0.077
M6	r_1, r_2, u_1, u_2 logs	0.714	1.000	0.510	0.076	1.000	-0.924
M7	All continuous core w/o Quantity Margin	0.714	1.000	0.510	0.077	1.000	-0.923
M26	GlobalLogX & GlobalLogY thresholds	0.713	0.996	0.510	0.027	-2.539	0.027
M2	NumLogDiff & UnitLogDiff	0.712	1.000	0.507	0.063	1.000	-0.937
M23	Number thresholds	0.552	0.607	0.501	-0.236	-793.764	–
M4	Left-unit normalized	0.701	1.000	0.492	0.052	1.000	-0.948
M5	Right-unit normalized	0.695	1.000	0.483	0.028	1.000	-0.972
M3	GlobalLogX & GlobalLogY	0.692	1.000	0.478	0.035	1.000	-0.965
M1	Quantity Margin only	0.690	1.000	0.476	0.022	1.000	-0.978
M27	XAsYUnitLog & YAsXUnitLog thresholds	0.626	0.951	0.413	-0.359	-91.628	–
M19	X-as-Y-unit thresholds	0.569	0.886	0.365	-0.270	-133.449	–
M17	GlobalLogX thresholds	0.488	0.661	0.361	-0.279	-646.158	–
M20	Y-as-X-unit thresholds	0.555	0.888	0.346	-0.353	-122.909	–
M13	r_1 thresholds	0.339	0.359	0.321	-0.298	-735.111	–
M14	r_2 thresholds	0.316	0.363	0.276	-0.361	-769.346	–
M18	GlobalLogY thresholds	0.426	0.665	0.273	-0.485	-647.091	–
M24	Unit-scale thresholds	0.376	0.698	0.202	-0.480	-690.965	–
M11	Unit identity	0.376	0.698	0.202	-0.464	-718.309	–
M12	Unit-pair identity	0.377	0.709	0.200	-0.437	-587.789	–
M22	UnitLogDiff thresholds	0.377	0.709	0.200	-0.471	-587.098	–
M9	Signed UnitLogDiff only	0.289	0.506	0.165	-0.734	-1079.523	–
M15	u_1-scale thresholds	0.244	0.409	0.146	-0.386	-791.680	–
M16	u_2-scale thresholds	0.196	0.409	0.094	-0.524	-842.436	–
M0	Intercept only	0.000	0.000	0.000	-0.004	-0.001	–

Table 8: Detailed surrogate results for the Metric length (Ex. m vs. cm) setting in Qwen3-4B-Base. R^2_{LM} measures how well each feature set predicts the LM's log-probability margin, while R^2_{Rule} measures how well it predicts Quantity Margin. Pred denotes Predictivity. For all examples, $\text{Pred} = R^2_{\text{LM}} \times (R^2_{\text{LM}}/R^2_{\text{Rule}})$. For the selected subset ($|\text{Quantity Margin}| \leq 0.2$), $\text{Pred} = R^2_{\text{LM}} - \max(R^2_{\text{Rule}}, 0)$ when $R^2_{\text{LM}} > 0$, and is left undefined otherwise. Rows are sorted by all-example Pred in descending order.

Feature set	All examples			$\lvert$Quantity Margin$\rvert \leq 0.2$		
	R^2_{LM}	R^2_{Rule}	Pred	R^2_{LM}	R^2_{Rule}	Pred
Signed NumLogDiff & UnitLogDiff	0.810	0.784	0.836	0.710	-249.479	0.710
All heuristic features	0.882	0.999	0.778	0.693	-1.050	0.693
Quantity Margin + all heuristics	0.882	1.000	0.777	0.694	1.000	-0.306
NumLogDiff & UnitLogDiff thresholds	0.822	0.995	0.680	0.643	-7.564	0.643
Primitive component thresholds	0.675	0.995	0.458	0.001	-7.975	0.001
NumLogDiff & UnitLogDiff	0.659	1.000	0.435	0.026	1.000	-0.974
Quantity Margin only	0.647	1.000	0.418	0.021	1.000	-0.979
GlobalLogX & GlobalLogY	0.646	1.000	0.418	0.017	1.000	-0.983

Table 9: Representative surrogate results for Imperial length (Ex. feet vs. inch) setting. R^2_{LM} measures how well each feature set predicts the LM's log-probability margin, while R^2_{Rule} measures how well it predicts the ground-truth comparison rule. Pred denotes Predictivity. For all examples, $\text{Pred} = R^2_{\text{LM}} \times (R^2_{\text{LM}}/R^2_{\text{Rule}})$. For the selected subset ($|\text{Quantity Margin}| \leq 0.2$), $\text{Pred} = R^2_{\text{LM}} - \max(R^2_{\text{Rule}}, 0)$ when $R^2_{\text{LM}} > 0$, and is left undefined otherwise. Rows are sorted by all-example Pred in descending order.

Feature set	All examples			$\lvert$Quantity Margin$\rvert \leq 0.2$		
	R^2_{LM}	R^2_{Rule}	Pred	R^2_{LM}	R^2_{Rule}	Pred
All heuristic features	0.938	0.999	0.881	0.658	-0.883	0.658
Quantity Margin + all heuristics	0.938	1.000	0.879	0.658	1.000	-0.342
Signed NumLogDiff & UnitLogDiff	0.778	0.735	0.823	0.050	-128.596	0.050
NumLogDiff & UnitLogDiff thresholds	0.884	0.995	0.785	0.375	-6.901	0.375
Primitive component thresholds	0.836	0.995	0.703	0.143	-10.171	0.143
NumLogDiff & UnitLogDiff	0.818	1.000	0.670	0.030	1.000	-0.970
GlobalLogX & GlobalLogY	0.804	1.000	0.646	-0.021	1.000	–
Quantity Margin only	0.803	1.000	0.646	-0.022	1.000	–

Table 10: Representative surrogate results for the Metric-Imperial length (Ex. m vs. feet) setting. R^2_{LM} measures how well each feature set predicts the LM's log-probability margin, while R^2_{Rule} measures how well it predicts the ground-truth comparison rule. Pred denotes Predictivity. For all examples, $\text{Pred} = R^2_{\text{LM}} \times (R^2_{\text{LM}}/R^2_{\text{Rule}})$. For the selected subset ($\lvert\text{Quantity Margin}\rvert \leq 0.2$), $\text{Pred} = R^2_{\text{LM}} - \max(R^2_{\text{Rule}}, 0)$ when $R^2_{\text{LM}} > 0$, and is left undefined otherwise. Rows are sorted by all-example Pred in descending order.

Feature set	All examples			$\lvert$Quantity Margin$\rvert \leq 0.2$		
	R^2_{LM}	R^2_{Rule}	Pred	R^2_{LM}	R^2_{Rule}	Pred
All heuristic features	0.851	0.998	0.725	0.749	-2.399	0.749
Quantity Margin + all heuristics	0.851	1.000	0.724	0.756	1.000	-0.244
Signed NumLogDiff & UnitLogDiff	0.724	0.762	0.689	0.723	-450.660	0.723
NumLogDiff & UnitLogDiff thresholds	0.735	0.997	0.543	0.741	-9.718	0.741
Primitive component thresholds	0.566	0.996	0.321	-0.010	-2.350	–
NumLogDiff & UnitLogDiff	0.553	1.000	0.306	-0.005	1.000	–
GlobalLogX & GlobalLogY	0.518	1.000	0.268	0.016	1.000	-0.984
Quantity Margin only	0.512	1.000	0.262	0.016	1.000	-0.984

Table 11: Representative surrogate results for the Metric mass (Ex. g vs. kg) setting. R^2_{LM} measures how well each feature set predicts the LM's log-probability margin, while R^2_{Rule} measures how well it predicts the ground-truth comparison rule. Pred denotes Predictivity. For all examples, $\text{Pred} = R^2_{\text{LM}} \times (R^2_{\text{LM}}/R^2_{\text{Rule}})$. For the selected subset ($\lvert\text{Quantity Margin}\rvert \leq 0.2$), $\text{Pred} = R^2_{\text{LM}} - \max(R^2_{\text{Rule}}, 0)$ when $R^2_{\text{LM}} > 0$, and is left undefined otherwise. Rows are sorted by all-example Pred in descending order.

Feature set	All examples			$\lvert$Quantity Margin$\rvert \leq 0.2$		
	R^2_{LM}	R^2_{Rule}	Pred	R^2_{LM}	R^2_{Rule}	Pred
Signed NumLogDiff & UnitLogDiff	0.785	0.768	0.801	0.635	-177.674	0.635
All heuristic features	0.887	0.998	0.788	0.740	-1.096	0.740
Quantity Margin + all heuristics	0.887	1.000	0.786	0.741	1.000	-0.259
NumLogDiff & UnitLogDiff thresholds	0.815	0.993	0.668	0.591	-6.336	0.591
Primitive component thresholds	0.719	0.993	0.521	0.091	-8.106	0.091
NumLogDiff & UnitLogDiff	0.691	1.000	0.478	0.032	1.000	-0.968
GlobalLogX & GlobalLogY	0.677	1.000	0.458	0.015	1.000	-0.985
Quantity Margin only	0.675	1.000	0.456	0.010	1.000	-0.990

Table 12: Representative surrogate results for the Metric-Imperial mass (Ex. g vs. ounce) setting. R^2_{LM} measures how well each feature set predicts the LM's log-probability margin, while R^2_{Rule} measures how well it predicts the ground-truth comparison rule. Pred denotes Predictivity. For all examples, $\text{Pred} = R^2_{\text{LM}} \times (R^2_{\text{LM}}/R^2_{\text{Rule}})$. For the selected subset ($\lvert\text{Quantity Margin}\rvert \leq 0.2$), $\text{Pred} = R^2_{\text{LM}} - \max(R^2_{\text{Rule}}, 0)$ when $R^2_{\text{LM}} > 0$, and is left undefined otherwise. Rows are sorted by all-example Pred in descending order.